MULTIVARIATE RISK AND OPTIMUM ECONOMIC ANALYSIS IN MARITIME STRUCTURES

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ABSTRACT

Multivariate risk analysis is sketched in general terms and applied to the two modalities of static failure: rigid and deformable. Estimates, programs and some risk and economic results for jetties on piles - Port of Sagunto - and rubblemound breakwaters - Port of Bilbao - are shown here.

INTRODUCTION

Since the introduction of the statistical methods in the study of the sea, Longuet-Higgins (1952), Pierson (1952), Saville (1953), an optimum economic design is being used in maritime works.

In the project of the rubblemound breakwater of Isla Rincon, Blume and Keith (1959) introduce the return period of the design storm concept and consider an optimum maintenance cost of the mantle.

Van der Kreek and Paape (1964) methodized the optimum economic design applicable to the mantle of rubblemound breakwaters.

Bores (1968) modified the damage probability proposed by Van der Kreek and Paape (1964) and, by using the probabilistic models obtained by Borgman (1963), methodized the optimum economic design for rigid structures and for the complete rubblemound breakwater, the mantle as well as the rest of the structure.

An optimum univariate design, with variables other than the wave height, has been proposed by Van Dantzig and Kriens (1960), Watt and Wilson (1974) and others.

In fact, however, the action of the sea on maritime works and structures depends not on one, but on several variables, whose effect on the structure should

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be analysed jointly.

Maritime, as well as terrestrial, works, structures, etc. must be designed to resist the resultant action of all variables, but not all of them in their extreme conditions. With the exception of very special cases, as in the design of nuclear power stations, certain military installations, etc. in which maximum security is required, it is not logical that all variables be considered extremal. When one variable occurs in extreme conditions - the principal variable - it is probable that the remaining variables have moderate values, perfectly determined by their mean annual distributions.

MULTIVARIATE RISK ANALYSIS

The static failure of strength, stability, sea room, etc. of any maritime structure, work, etc. to the wind wave action always occurs according to two modalities, Bores (1968). Lee (1962) proposes three. In the first complete failure is originated by the action of one wave alone. In the second failure is produced by the action of several separate waves.

With the second modality there exists a zone of partial failure, where the damage depends on the magnitude of the variables. In fact, each wave whose magnitude exceeds those corresponding to zero damage will originate a damage on the structure. These waves are always possible in the case of wind waves, and their probability of occurrence depends on the number of waves, on the persistence, and the energy of the state of the sea.

In each particular case, wave variables - height, period, steepness, etc. - can be related with those of the earthquakes, wind, tides - astronomical and meteorological - forces due to ships, vehicles, harbour facilities, etc. and with those of the structure and soil.

The multivariate characteristic function

$$\kappa(v_1, \ldots, v_j, \ldots) = 0 \qquad (1)$$

forms, for given values of the structure and soil parameters, a hypersurface which separates levels of damage. In the case of failure of the first modality, rigid failure, there is only one level marking the boundary between stability and instability.

In the second modality, which we call deformable failure, the number of possible levels depends on the accuracy of the results required in each particular case. Ten per cent damage can be an adequate step for the rubblemound breakwater mantle.

Variables can be free or constrained. Some variables can be independent for a given state of the sea, but if all of them depend on it, they will be constrained - H and T of wind waves, for example.

The probability of a given level of damage in deformable failure, or colapse in rigid failure, can be written by means of the Lebesque-Stieltjes integral

(2) $$p = \int_{\kappa} p(v_1,..,v_j,..)\, dg(v_1)..dg(v_j)..$$

where κ, the multivariate characteristic function, is the limit of integration.

If all variables are independent the expression (2) will take the form

(3) $$p = \int \Big| .. \int \Big| \int_{\kappa} p(v_1,..,v_j,..)\, dg(v_1) \Big| dg(v_2) ... \Big| dg(v_j)..$$

which can be integrated step by step.

RIGID FAILURE: JETTIES ON PILES

An estimate of the multivariate characteristic function for this particular case can be written in the form

(4) $$\kappa \left| \begin{matrix} Z_0, I, E, B \ ;\ D, \sigma_B, \sigma_S, \ Q \ ;\ C_D, C \ , \rho_W \ ;\ F \ ; \\ H, T, N, S_{AT}, S_{MT} \ ;\ \rho_{\sigma_B}, \rho_{\sigma_S}, \rho_H, \rho_T, \rho_N, \rho_S, \rho_F, \rho_\Sigma \end{matrix} \right| = 0$$

where, Z_0, I are the depth and the slope of the bottom
E , B the coefficients of embedding and berth
D the diameter of the pile

σ_B, σ_S the concrete and steel strengths

C_D, C_I the drag and inertial coefficients

ρ_W the density of the water

H,T,N the wave height, period and persistence of the wind waves

S_{AT}, S_{MT} the astronomical and meteorological tide setup

F the loads

$\rho_H \cdot\cdot \rho_\Sigma$ risk coefficients for wave height, periods, persistence, astronomical and meteorological tide setup, loads and scattering of the tests.

Dealing with a rigid failure, the characteristic wave height will be the probable maximum wave height occurring in the expected life of the structure. In fact, this wave can occur with any state of the sea, however low it may be, Freudenthal (1968), (1969), but the probability that this may happen is very small. Only with states of the sea of high energy, storms, is it probable that such a wave height occurs, in consequence, a good estimate can be obtained by considering only these states of the sea. The probable maximum wave height is in this case a constrained variable that can be written in the form, Bores (1968),

(5) $$H_{max;N} = H_{1/3}\{(\ln N)/2\}^{1/2}$$

Sea level, S, depends on a number of components: astronomical tide, pressure and wind setup or meteorological tide, tsunamis, resonance waves on the coastal platform and/or basin, etc. In this case we have considered only the most important and best known phenomena occurring on our coasts, the astronomical tide and the meteorological tide and we admit the Boltzman´s Principle of linear superposition. Consequently the sea levels must be considered as a constrained variable that we can write in the form

(6) $$S = S_{AT} + S_{MT}$$

All these variables are random and their corresponding distributions of probability, regimes, can be estimated from wave or level forecasting methods or recordings.

For given values of the bottom, loads and structure parameters, the joint probability that the climatic random variables exceed the characteristic hypersurface is

(7) $$p = \iint_K p(H_k)\, dp_T\, dp_S$$

which can be written in the form

(8) $$p = \sum_{ij} \Pr(H_k \geq H_{k_{2i+1,2j+1}})\ \Delta p_{T_{2i+1}}\ \Delta p_{S_{2j+1}}$$

for numerical integration.

In the present state of art we admit that all the variables are independent and T free, therefore

(9) $$\Delta p_{T_{2i+1}} = \Pr(T_{2i} \geq T \geq T_{2i+2}) = \Pr(T \geq T_{2i}) - \Pr(T \geq T_{2i+2})$$

As we have seen, the characteristic wave height is a constrained variable, related with the significant wave height, $H_{1/3}$, and the persistence, N, by means of the expression (5), then

(10) $$\Pr\left| H_k \geq H_{k_{\substack{2i+1\\2j+1}}} \right| = \sum_m \Pr\left| H_k \geq H_{k_{\substack{2i+1\\2j+1\\2m+1}}} \right| \Delta p_{N_{2m+1}}$$

where

(11) $$\Delta p_{N_{2m+1}} = \Pr(N_{2m} \leq N \leq N_{2m+2}) = \Pr(N \geq N_{2m}) - \Pr(N \geq N_{2m+2})$$

and

$(5)_b$ $$H_k = H_{1/3}\{(\ln N)/2\}^{1/2}$$

In analogous way

(12) $$\Delta p_{S_{2j+1}} = \Pr(S_{2j} \leq S \leq S_{2j+2}) = \Pr(S \geq S_{2j}) - \Pr(S \geq S_{2j+2})$$

but

(13) $$Pr(S \geq S_{2j}) = \sum_n Pr(S_{MT} \geq S_{2j,2n+1}) \, \Delta p_{S_{AT_{2n+1}}}$$

and

(14) $$Pr(S \geq S_{2j+2}) = \sum_q Pr(S_{MT} \geq S_{2j+2,2q+1}) \, \Delta p_{S_{AT_{2q+1}}}$$

where

(15) $$\Delta p_{S_{AT_{2n+1}}} = Pr(S_{AT_{2n}} \leq S_{AT} \leq S_{AT_{2n+2}}) = Pr(S_{AT} \geq S_{AT_{2n}}) - Pr(S_{AT} \geq S_{AT_{2n+2}})$$

and

(16) $$\Delta p_{S_{AT_{2q+1}}} = Pr(S_{AT_{2q}} \leq S_{AT} \leq S_{AT_{2q+2}}) = Pr(S_{AT} \geq S_{AT_{2q}}) - Pr(S_{AT} \geq S_{AT_{2q+2}})$$

PORT OF SAGUNTO (*)

An application of the above methodology has been realized in the Port of Sagunto, Spain.

No interaction among piles nor dynamic effects have been considered in this case and hydrodynamic forces used are the maximum obtained by Reid and Bretschneider (1959).

Estimates of wave height and period distributions were obtained from wave forecasting - Integrated Method, Bores (1967) - of the highest annual storms - Direct Method, Bores (1969),(1974) - occurring during the period 1963-75. Wave persistence, astronomical and meteorological tide distributions were estimated. Wave characteristics have been limited by wave breaking, due to the bottom and/or form effects.

Flowchart of the Program - in Basic for an IBM 5100 Computer - and multivariate risk for $Z_0 = 20$ m., $I = 0{,}01$; $E = 2D$; $B = 0{,}75D$; $W = 500$ tons; $\sigma_B = 2000$ ton/m^2, $Q = 0{,}5$; $C_D = 1{,}7$; $C_I = 2$; $\gamma = 1{,}03$ ton/m^3; risk coeff. equal to the unity, are shown in fig. 1 and 2.

Figure 3 shows the intersection of the multivariate characteristic function, a hypersurface, as indicated above, with the plane $S = 0$.

(*) Results here included are part of the study requested of the author by "Cubiertas y Tejados S.A." in May 76 and presented in the Spanish Civil Eng. Professional College in October 76.

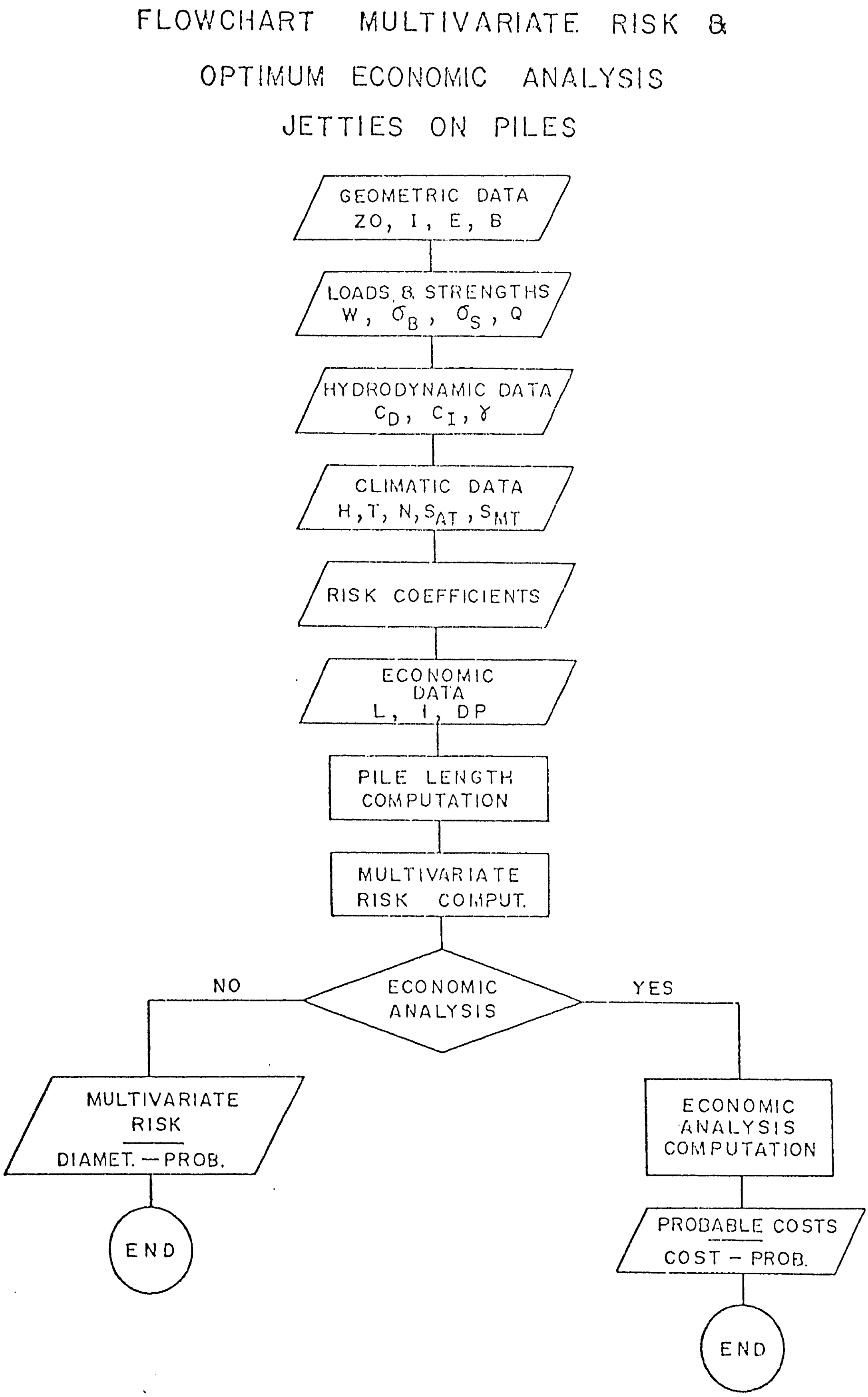
FLOWCHART MULTIVARIATE RISK &
OPTIMUM ECONOMIC ANALYSIS
JETTIES ON PILES
GEOMETRIC DATA
ZO, I, E, B
LOADS & STRENGTHS
W, σB, σS, Q
HYDRODYNAMIC DATA
CD, CI, γ
CLIMATIC DATA
H, T, N, SAT, SMT
RISK COEFFICIENTS
ECONOMIC DATA
L, I, DP
PILE LENGTH COMPUTATION
MULTIVARIATE RISK COMPUT.
ECONOMIC ANALYSIS
NO
YES
MULTIVARIATE RISK
DIAMET. — PROB.
END
ECONOMIC ANALYSIS COMPUTATION
PROBABLE COSTS
COST — PROB.
END

Fig. 1

MULTIVARIATE RISK ANALYSIS

SAGUNTO

MULTIVARIATE ECONOMIC ANALYSIS

DEPTH	20	COEFF. CD	1.7
LOAD	500	R6	6E-2
SLOPE	1E-2	R8	.15
D1	1	APPROX. $	1E-2

DIAMETER	PROB. P
1	.877
1.1	.274
1.2	.105
1.3	5.474E-2
1.4	3.162E-2
1.5	2.092E-2
1.6	1.409E-2
1.7	9.870E-3
1.8	7.356E-3
1.9	5.356E-3
2	4.056E-3
2.1	2.744E-3
2.2	1.258E-3

Fig.2

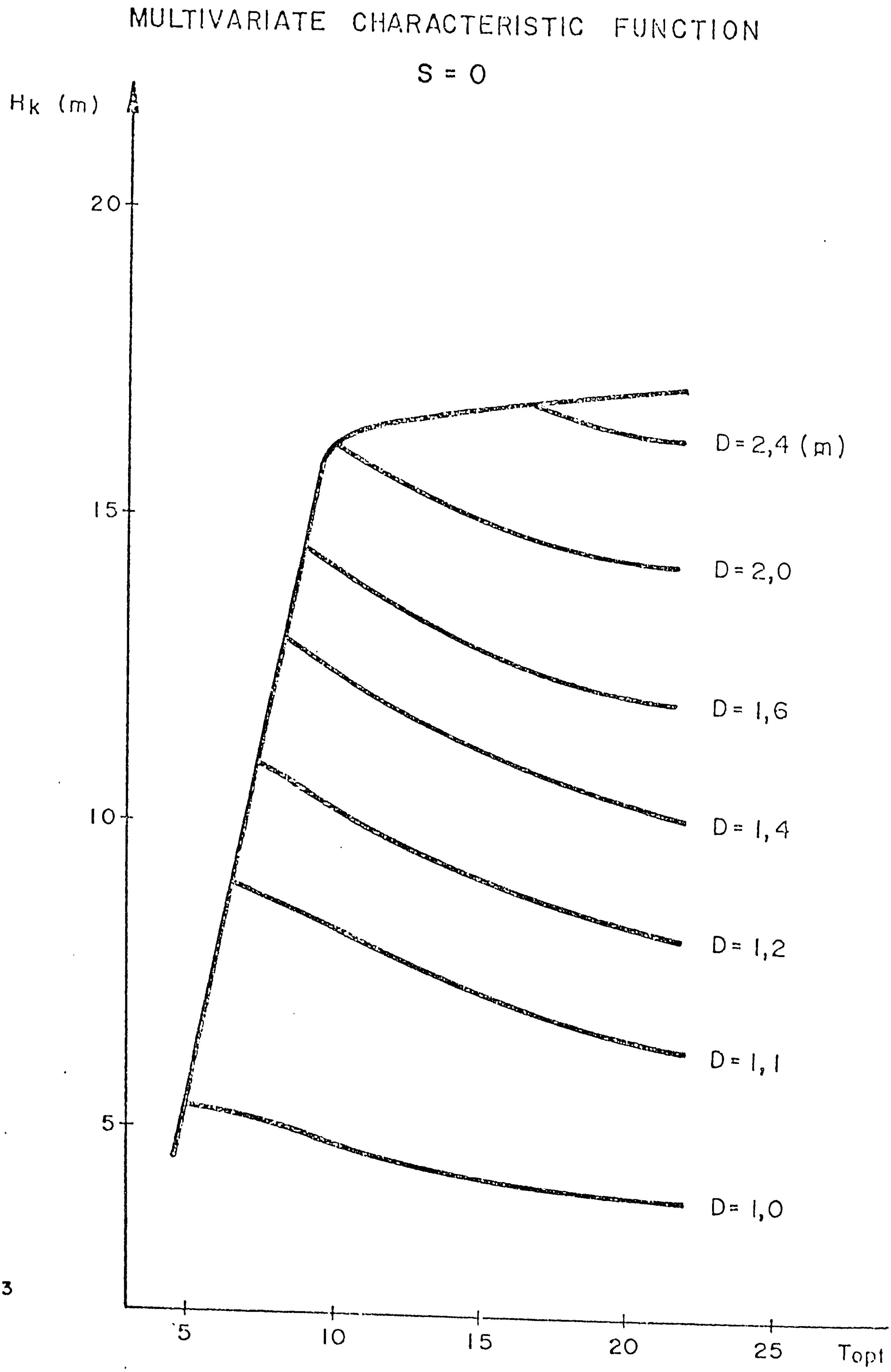
MULTIVARIATE CHARACTERISTIC FUNCTION
S = 0
Hk (m)
20
15
10
5
D = 2,4 (m)
D = 2,0
D = 1,6
D = 1,4
D = 1,2
D = 1,1
D = 1,0
5
10
15
20
25
Topt

Fig. 3

DEFORMABLE FAILURE: RUBBLEMOUND BREAKWATERS

The stability of the blocks forming the main mantle of a rubblemound breakwater depends on a number of parameters - specific gravity, introduced by Castro (1934), slope of the mantle, Iribarren (1938), Hedar (1963), Iribarren (1965), macro and micro geometry of the blocks and the mantle, Iribarren (1938), Hudson (1953) - and the climatic variables - wave height, Castro (1934), Iribarren (1938), period and/or steepness, Ahrens (1974), Bruun (1976).

Placed at random in two or three layers on a secondary mantle, formed also with loose elements of drastically smaller size, the blocks of the main mantle of a rubblemound breakwater form a layered granular set, whose elements are not in the same stability conditions. Some of the upper, or superjacent, blocks are simply leaned on the other blocks and they can be removed by hydrodynamic forces, turning in their places without any movement of the other blocks. The infrajacent blocks support the weight of the superjacent and/or other infrajacent ones and in such a situation their dislodging by waves, no matter how high, is impossible, unless the slope is too steep, in which case the complete mantle can slip down as a whole.

Rubblemound breakwaters are one of the best examples of deformable failure, and damage can evolve on them from zero until the complete disappearance of a part of the main mantle, the active mantle, Iribarren (1965). In this situation, with the secondary mantle exposed to the direct attack of the waves, damage progresses quickly, due to the small weight of the blocks of the secondary mantle, and other phenomena, not hydrodynamic, begins to contribute to the breakwater colapse. Parts of the main and/or secondary mantle can creep or slide down and the screen can tilt and fall over seaward because of base failure.

For a given slope of the mantle, the critical stability conditions of the superjacent blocks are not identical, because each particular block, placed at random, has its angle of tilting and it is affected by a hydrodynamic flux which depends on the instantaneous local configuration of the mantle.

Stability of the blocks is improved by attacking the breakwater with small waves which by slightly moving the blocks interlock them in more stable positions.

When damage progresses, the removal of blocks changes the slope of the mantle in the neighborhood of the mean water level, in the active mantle, and tends to form more stable angles. Hedar (1963) and Iribarren (1965) found with monocromatic waves a critical slope for each particular form of the blocks, separating up and downslope equilibrium.

The ultimate strength of a rubblemound breakwater against wave action depends, then, on the stability conditions of the more interlocked blocks of the active mantle and also on the critical slope.

Between these two limits, ultimate and zero damage, rubblemound breakwaters display a zone of partial stability. Beyond the upper limit no block can resist wave action and rubblemound breakwaters become instable.

Due to the random nature of the granular set, even with monocromatic waves, scattering of damage exists, but in a given breakwater attacked by monocromatic waves of magnitudes comprised between those corresponding to these limits, damage only progresses until a certain stage, damage is limited, since some blocks are interlocked in more stable positions.

Very different is the case of wind waves, since, as we said above, waves with any magnitude are possible, and their probability of occurrence depends only on the persistence and on the energy of the state of the sea.

Since any wave whose magnitudes exceed those corresponding to a given stage of damage is active, that is, colaborates to produce damage, the characteristic wave height in this case must be written, Bores (1968),

$$\frac{H_{1/n}}{\sqrt{E}} = 2(\ln n)^{1/2} + n\sqrt{\pi}\,\{1 - \phi|(\ln n)^{1/2}|\} \tag{17}$$

where ϕ is the normal distribution of probability and E the cumulative energy of the spectrum, Longuet-Higgins (1952).

As in the above application, sea level, S , must be considered as a constrained variable of the form

$$S = S_{AT} + S_{MT} \tag{18}$$

An estimate of the multivariate characteristic function for rubblemound breakwaters can then be written in the form

(19) $$\kappa \left| \begin{matrix} Z_0, I \;;\; W, \gamma_b, f, K \;;\; \alpha, C, B \;;\; D \\ H, T, N, S_{AT}, S_{MT} \;;\; \rho_H, \rho_T, \rho_N, \rho_S, \rho_\Sigma \end{matrix} \right| = 0$$

where,

Z_0, I	are the depth and the slope of the bottom
W, γ_b	the weight and specific gravity of the blocks
f, K	block parameters
α	slope of the mantle
B, C	mantle parameters
H, T, N	wave height, period and persistence of the wind waves
S_{AT}, S_{MT}	astronomical and meteorological tide setup
$\rho_H \cdot\cdot \rho_\Sigma$	risk coefficients for wave height... and scattering of the tests'

For given values of the bottom, mantle and block parameters and for each stage of damage, usually stated in percentage of the mantle, or the active mantle, the joint probability that the climatic random variables exceed the multivariate characteristic hypersurface, κ, is

(20) $$p = \iiint_\kappa p(H_{1/n})\; dp_N\; dp_T\; dp_S$$

which can be written in the form

(21) $$p = \sum_{ijk} \Pr \left| H_{1/n} \geq H_{1/n_{\substack{2i+1\\2j+1\\2k+1}}} \right| \Delta p_{N_{2i+1}} \Delta p_{T_{2j+1}} \Delta p_{S_{2k+1}}$$

In the present state of art we admit that all variables are independent and N and T free, therefore

(22) $$\Delta p_{N_{2i+1}} = \Pr(N_{2i} \leq N \leq N_{2i+2}) = \\ = \Pr(N \geq N_{2i}) - \Pr(N \geq N_{2i+2})$$

(23) $$\Delta p_{T_{2j+1}} = \Pr(T_{2j} \leq T \leq T_{2j+2}) = \\ = \Pr(T \geq T_{2j}) - \Pr(T \geq T_{2j+2})$$

and

$$\Delta p_{S_{2k+1}} = Pr(S_{2k} \leq S \leq S_{2k+2}) = \tag{24}$$
$$= Pr(S \geq S_{2k}) - Pr(S \geq S_{2k+2})$$

with the values of $Pr(S \geq S_{2k})$ and $Pr(S \geq S_{2k+2})$ given in (13) to (16).

PORT OF BILBAO

A complete application of the methodology here introduced is in progress for the rubblemound breakwaters of the Port of Bilbao. Results briefly presented here correspond to a first estimate (#) in which only the wave height and persistence have been considered.

The estimate of the wave height extremal distribution has been obtained from wave forecasting, by means of the Integrated Method, Bores (1967), of the highest annual storms - Direct Method, Bores (1969), (1974) - ocurring during the period 1951/71.

The persistence distribution has been estimated from real storm recording, obtained by means of two Datawell buoys in operation during the last two years, computing the number of waves in each storm that exceed the characteristic wave height, $H_{1/10}$, corresponding to that storm maximum.

The corresponding distributions, regimes, obtained are

$$H_{1/10} = 5.15 + 1.665\ y_H \tag{25}$$

$$\nu = 109.26 + 27.32\ y_\nu \tag{26}$$

(#) Results here included are a part of the study requested of the author by the Harbor Authority of Bilbao in August 1976 and submitted to the Harbor Authority and to the general contractor, the group "Dragados & Construcciones S.A.","Intecsa", in October 1976. Some lectures on these results and on the multivariate risk and optimum economic analysis have also been given this fall by the author in his class of Ports at the University.

The multivariate characteristic function has been estimated by admitting the following hypothesis:

1 - Every wave higher than that of zero damage is active.

2 - All active waves affect the mantle in the same way, independently of their height.

3 - The number of active waves necessary for the complete colapse of the main mantle depends on the number of its layers. For two layers we estimate 300 wavės, Bores (1968).

4 - Damage is a monotonous increasing function of the persistence, number of active waves. A linear, conservative, hypothesis is admited in this first report.

Tests iniciated in the Danish Hydraulic Institute in February and still in progress support this first estimate.

Economic analysis is realized as proposed by Van der Kreek and Paape (1965) and modified by Bores (1968), changing the univariate for the corresponding multivariate probability.

Flowchart of the Program - in Basic for an IBM 5100 Computer - and multivariate optimum economic analysis for $\gamma_b = 2.4$; $f = 2.84$; $K = .452$; $\cot \alpha = 1.8$; $D = .1$; $W = 10$ tons ; $\rho_\Sigma = 1.1$; expected life of the breakwater $E = 25$ years ; interest $i = .1$; defended property = 1000000 ptas/m ; repair increment of the mantle = 3 ; concrete price = 3000 ptas/m3 ; void index = .4 ; are shown in figures 4 and 5.

Figure 6 shows the multivariate characteristic function. Crosses indicate the percentage of damage in terms of the active mantle volume, deduced from the Danish Hydraulics Institute tests. Numbers parenthetically represent the number of waves.

CONCLUSIONS

1 - A general methodology for Multivariate Risk and Optimum Economic Analysis is introduced and the corresponding general multivariate characteristic function and probability of exceedence are outlined.

2 - Application to rigid and deformable structures is made in the particular cases of jetties on piles and

FLOWCHART MULTIVARIATE RISK &

OPTIMUM ECONOMIC ANALYSIS

RUBBLEMOUND BREAKWATERS

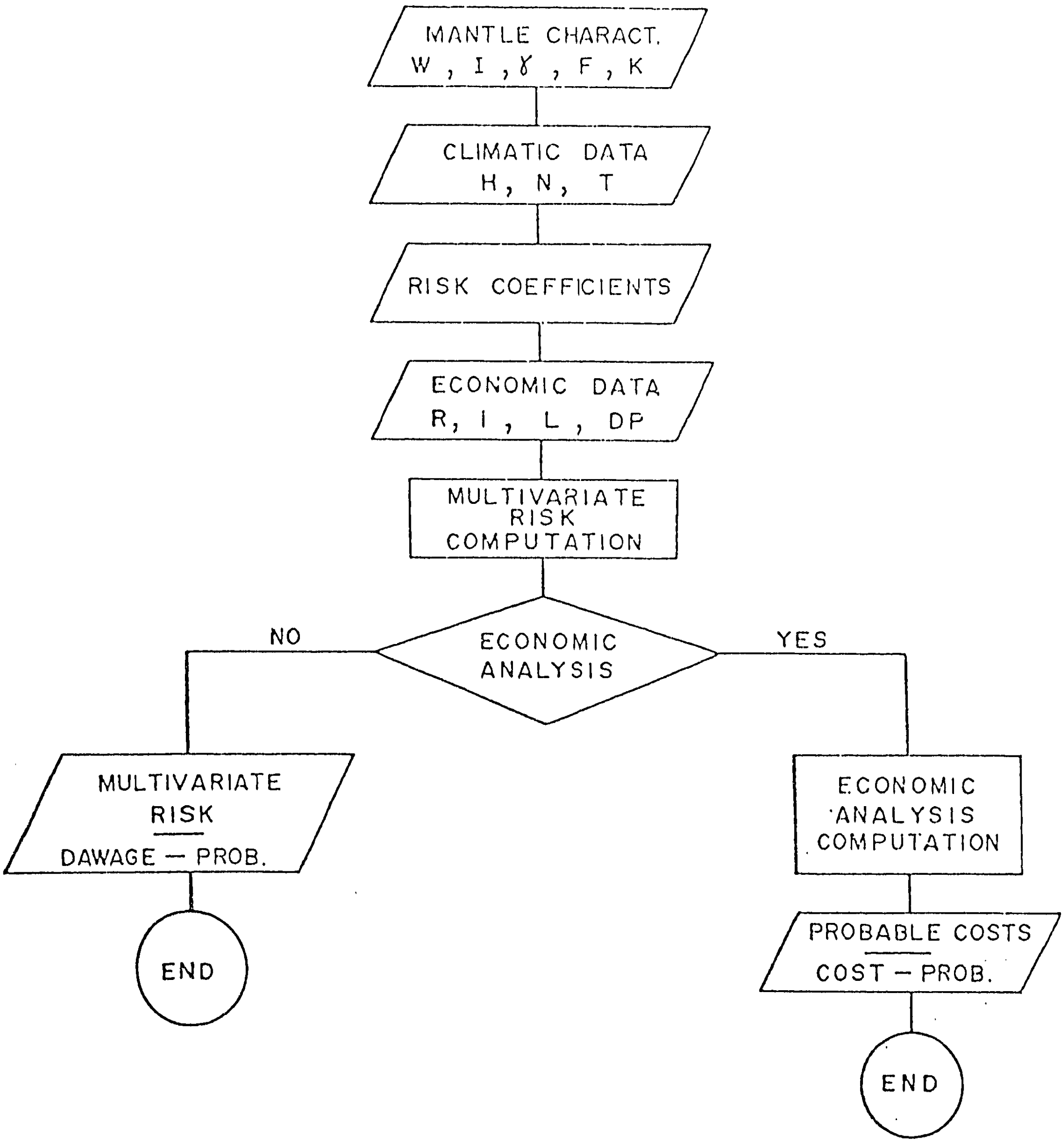

Fig. 4

MULTIVARIATE OPTIMUM ECONOMIC ANALYSIS

BILBAO

MALTIVARIATE ECONOMIC ANALYSIS

BLOCK WEIGHT	50	RISK COEFF.	1.1
SPECIFIC GRAVITY	2.4	APPROX. $	.5
SLOPE	1.8	APPROX. D3/D2	.1

WEIGHT	TOTAL COST	PROB. ZERO DAMAGE
50	3.545651	.297999
60	3.247429	.239209
70	3.071330	.195153
80	2.968730	.161438
90	2.912987	.135157
100	2.880454	.114339
110	2.885430	9.761432E-2
120	2.897633	8.400903E-2
130	2.920846	7.281908E-2
140	2.952156	6.352494E-2

Fig. 5

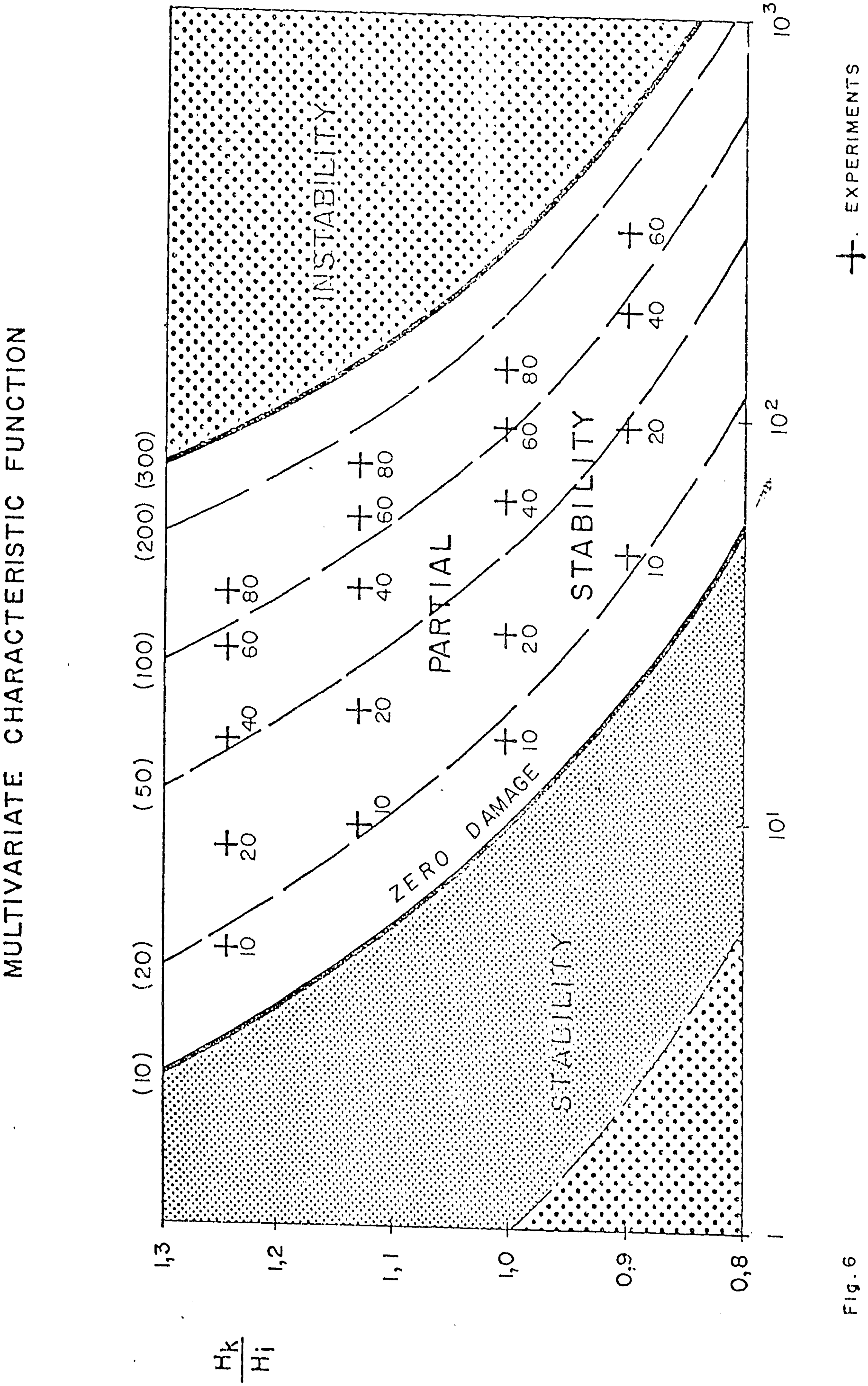
MULTIVARIATE CHARACTERISTIC FUNCTION
INSTABILITY
PARTIAL STABILITY
ZERO DAMAGE
STABILITY
EXPERIMENTS
(10) (20) (50) (100) (200) (300)
1,3 1,2 1,1 1,0 0,9 0,8
1 10^1 10^2 10^3
$\frac{H_k}{H_I}$

Fig. 6

rubblemound breakwaters. Programs in Basic for an IBM 5100 Computer are operative for these two cases.

3 - Estimates for the Port of Sagunto and the Port of Bilbao are included.

4 - Investigation is in progress on the above cases and on vertical breakwaters for which the corresponding program is also operative and tests are being iniciated (+).

(+) A study for the Port of Bilbao has been requested of the author by the group "Dragados & Construcciones S.A.","Intecsa", in March 1977 and the corresponding tests have already been contracted with the British Hovercraft Corporation Laboratories.

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